Let $\boldsymbol{x}_n = \begin{pmatrix} p_n & q_n & r_n \end{pmatrix}^T$ (initial condition $\boldsymbol{x}_0 = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}^T$). The transition can be noted as:

$$A = \begin{pmatrix} \frac{1}{2} & \frac{2}{5} & 0 \\ \frac{1}{2} & \frac{2}{5} & 0 \\ 0 & \frac{1}{5} & 1 \end{pmatrix}, \quad \boldsymbol{x}_n = A\boldsymbol{x}_{n-1} \quad (n \in \mathbb{Z}^+)$$

Observe the first two identical row vector. When $n \geq 2$, with $p_1 = q_1 = \frac{1}{2}$ we conclude that $p_n = q_n = \frac{1}{2}p_{n-1} + \frac{2}{5}q_{n-1} = \frac{9}{10}p_{n-1}$. Let $\boldsymbol{y}_n = \begin{pmatrix} p_n & r_n \end{pmatrix}^T$ (with $\boldsymbol{y}_1 = \begin{pmatrix} \frac{1}{2} & 0 \end{pmatrix}^T$). The equation is then shrunk into two dimension:

$$B = \begin{pmatrix} \frac{9}{10} & 0 \\ \frac{1}{5} & 1 \end{pmatrix}, \quad \boldsymbol{y}_n = B\boldsymbol{y}_{n-1} \quad (n \geq 2)$$

The question only focuses on the calculation of $r_n = \frac{1}{5}p_{n-1} + r_{n-1}$. Note that $\{p_n\}$ $(n \geq 1)$ decays geometrically with factor $\frac{9}{10}$.

Sum up two side of r_k equation, for $k \in [2, n]$:

$$r_n = \frac{1}{5}\sum_{k=1}^{n-1} p_k + r_1 = \frac{\cancel{1}}{\cancel{1}0} \times \frac{1 - \left(\frac{9}{10}\right)^n}{\cancel{1 - \frac{9}{10}}} = 1 - \left(\frac{9}{10}\right)^{n-1} \quad (n \geq 2)$$

Considering $n < 2$, the final result:

$$r_n = \begin{cases} 0 & \text{if } n < 2 \\ 1 - \left(\frac{9}{10}\right)^{n-1} & \text{otherwise} \end{cases}$$